Workshop on graphs, networks and brains

May 12-13, Rényi Institute

Monday

11am, László Barabási (CEU): **Physical Network Constraints Define the Lognormal Architecture of the Brain's Connectome** (Turan Seminar Room)

2.15pm, György Buzsáki (NYU): **Ways to think about the brain** (Rényi colloquium, Nagyterem)

Tuesday (20 minute focused talks, Tondos Seminar Room)

10.00 - Balázs Hangya (KOKI): **Reinforcement learning and the brain**

10.20 - György Buzsáki (NYU): **The nonlinear brain: from dynamics to cognition**

10.40 - 11.20 Coffee break

11.20 - Márton Pósfai (CEU): **Network dismantling by physical damage**

11.40 - Gábor Pete (Rényi): **Structural results for the Tree Builder Random Walk**

12.00 - Balázs Ujfalussy (KOKI): **Partial remapping enables efficient generalization for discrete and continuous variables**

12.20 - Lunch break

14.00 - Jun Yamamoto (CEU): **Reentrant Localizations in Physical Laplacian Eigenvectors**

14.20 - Ádám Timár (Rényi): **Local proper colorings of networks**

14.40 - 15.20 Coffee Break

15.20 - Miklós Abért (Rényi): **Degree biased random walks on degree biased preferential attachment graphs**

15.40 - Pierfrancesco Dionigi (Rényi): **Higher order preferential attachment models: mean field approaches**

16.00 - Jasper Van Der Kolk (CEU): **Network Design: Faithful Reproduction of Networked Systems**

Match the talk to the abstract (a little game):

The brain is certainly capable of both model based, supervised, explicit learning and model free, unsupervised, implicit learning. Indeed, finding a link between reinforcement learning theory and dopaminergic neuron activity in the primate brain was arguably among the most influential breakthroughs of theoretical neuroscience. But how good is this analogy? In other words, is this really how the brain learns? I will share somewhat loosely connected thoughts on the ups and downs of reward prediction error representation theories, including an army of multiple imperfect RL agents, and neural computations not necessarily carried out as expected. Is there such a thing as a 'unifying theory'?

Consider a network of routers, where if two routers are too close to each other, they must choose a different channel for communication to avoid interference. This problem can be rephrased as a vertex-coloring problem of the respective network. A natural requirement is that no central organiser is present, that is, each router has to decide which channel to choose, based on its information on the nearby routers and maybe using some randomness that it generates for itself. The overlapping of the neighborhoods may cause some dependence between the resulting local outputs, but how much of this dependence is necessary? Similar problems are addressed by distributed computing models, or by factor of iid problems in probability. We will present recent results on local proper colorings where the colors of two vertices are independent whenever the vertices are at distance at least 4.

The connectivity pattern of many networked systems, from the ribosome to the neurons of the flatworm C. elegans, are highly conserved between instances. Traditional network models like the configuration model are not able to explain this phenomenon, focusing on specific network properties rather than the exact wiring. In this work we introduce a framework for network design, providing the tools to understand how unique networks can be formed from a set of building blocks and predetermined connectivity rules.

The brain has long been conceptualized as a network of neurons connected by synapses. However, attempts to describe the connectome using established network science models have yielded conflicting outcomes, leaving the architecture of neural networks unresolved. Here, by performing a comparative analysis of eight experimentally mapped connectomes, we find that their degree distributions cannot be captured by the well-established random or scale-free models. Instead, the node degrees and strengths are well approximated by lognormal distributions, although these lack a mechanistic explanation in the context of the brain. By acknowledging the physical network nature of the brain, we show that neuron size is governed by a multiplicative process, which allows us to analytically derive the lognormal nature of the neuron length distribution. Our framework not only predicts the degree and strength distributions across each of the eight connectomes, but also yields a series of novel and empirically falsifiable relationships between different neuron characteristics. The resulting multiplicative network represents a novel architecture for network science, whose distinctive quantitative features bridge critical gaps between neural structure and function, with implications for brain dynamics, robustness, and synchronization.

We study the Tree Builder Random Walk: a randomly growing tree, built by a walker as she is walking around the tree. Namely, at each time n, she adds a leaf to her current vertex with probability $p_n \asymp n^{-\gamma}$, $\gamma\in (2/3,1]$, then moves to a uniform random neighbor on the possibly modified tree. We show that the tree process at its growth times, after a random finite number of steps, can be coupled to be identical to the Barabási-Albert preferential attachment tree model.
Thus, our TBRW-model is a local dynamics giving rise to the BA-model. The coupling also implies that many properties known for the BA-model, such as diameter and degree distribution, can be directly transferred to our TBRW-model, extending previous results.

We have been working on a version of preferential attachment that takes in account higher order statistics of the graph. This turns out to be a class of models with an interesting behavior different from the classic BA preferential attachment. So far it was not possible to reduce this class of models to a Pólya urn model. This prompted attempts to derive a satisfying description by simplified mean field approaches. I will describe these approaches and where we stand in the full understanding of these models.

We explore the robustness of complex networks against physical damage. We focus on spatially embedded network models and datasets where links are physical objects or physically transfer some quantity, which can be disrupted at any point along its trajectory. To simulate physical damage, we tile the networks with boxes of equal size and sequentially damage them. By introducing an intersection graph to keep track of the links passing through tiles, we systematically analyze the connectivity of the network and explore how the physical layout and the topology of the network jointly affect its percolation threshold. We show that random layouts make networks extremely vulnerable to physical damage, driven by the presence of very elongated links, and that higher-dimensional embeddings further increase their vulnerability. We compare this picture against targeted physical damages,showing that it accelerates network dismantling and yields non-trivial geometric patterns. Finally, we apply our framework to several empirical networks, from airline networks to vascular systems and the brain, showing qualitative agreement with the theoretical predictions.

During navigation, place cells in the rodent hippocampus form a map of the environment. The hippocampal code is also modulated by several non-spatial variables including the presence of specific sensory cues, objects, history or task. For a map to be useful the position should be represented independent of the content: a novel object or task change can modify the map, but ideally the changes in the code should still allow an invariant positional readout. The conditions for neuronal representations that generalise across different situations have been developed for categorical variables, such as context, stimulus or choice. However, the coding principles are fundamentally different for continuous variables and thus it is not known what kind of spatial representations enable efficient generalization across different maps.
Here we show that partial remapping allows hippocampal representations to incorporate novel content and yet to generalise the position code. We derive analytically that position coding remains robust to the remapping of a surprisingly large fraction of the neurons and use simulations to demonstrate that it tolerates heterogeneity in the coding properties of individual cells. Next we investigate partial remapping in tasks involving categorical variables and demonstrate that the majority of the neurons can change their tuning to task variables after context change without drop in the decoding performance. We also show that in simple cases partial remapping allows efficient generalization without reducing the dimensionality of the neuronal activity. Finally, we use 2P calcium imaging data to show that partial remapping is a common strategy used in the rodent hippocampus to represent context in spatial tasks.
Our findings explains how behavioral performance can remain intact in situations classified previously as global remapping. We suggest that partial remapping is a general strategy of the brain to multiplex context and content in neuronal representations.

We study degree power biased random walks on degree power biased preferential attachment graphs. It turns out that these systems admit several different phase

transitions, in terms of asymptotic orthogonality of stationary measures and the behavior of the random walk on the limit of the networks.
As an example, let BA be the limit of the Barabasi-Albert tree BA(n). This is a (random) one-ended rooted tree. We show that the simple random walk is recurrent on BA, but the linear degree biased random walk is transient on it. Interestingly, transience itself exhibits at least two different phases for this simple model: in one phase, the particle will not return to its origin but still gravitate to the ‘poorest’ nodes, in another phase, it will strongly converge to the unique limit point, meaning, towards the ‘very rich’ nodes.

Physical networks in various physical and biological systems exhibit degree-volume correlations. While it has been shown that such a correlation emerges naturally under a simple set of physical constraints, it remains unclear how the degree-volume correlations influence the dynamics on physical networks. In this study, we investigate the eigenvectors of physical Laplacian, i.e., a volume-weighted graph Laplacian, that captures diffusion-like dynamics on physical networks with distinct inter- and intra-physical node coupling strengths. In particular, we focus on the role of degree-volume correlation strength by analyzing localization patterns in the eigenvectors of the physical Laplacian. Our findings reveal that heterogeneity in node degree-to-volume ratios induces *reentrant localization* in both Fiedler and leading eigenvectors.